Journal of Alloys and Compounds, 211/212 (1994) 104–106
JALCOM 006

Nonlinear damping and harmonic generation in iron

D.N. Beshers and V.F. Coronel
Henry Krumb School of Mines, Columbia University, New York, NY 10027 (USA)

Abstract

The question of the phases of the third harmonic signals generated by dislocations is reconsidered. For an artificial model of breakaway, it is shown that the phase of the harmonic wave depends on the phase at which the breakaway takes place, and therefore on the amplitude of vibration. Reversals of sign of the harmonic phase can occur at certain phases of breakaway, leading to (partial) cancellation of waves from segments of different breakaway stress. This effect seems likely to be the major part of the phase cancellations previously reported and attributed, in part, to screw-edge cancellation.

1. Introduction

We have reported previously on the damping and harmonic generation of iron under a saturating magnetic field [1, 2]. The damping data for annealed iron, on Granato–Lücke (GL) plots, show two linear segments. The third harmonic, as a function of amplitude, showed a remarkable sharp dip at a strain amplitude of about 1.7×10^{-4}, a little below the upper limit of the first linear segment in the GL plot. We suggested that the two GL segments were to be attributed to edges (first) and screws, because it is well established that edges move more easily than screws in iron and other body-centered cubic metals, at least at relatively low temperatures. That edge and screw dislocations should emit third harmonic with opposite phase was predicted by Hikata and Elbaum (HE) [3], based on consideration of bowout without breakaway, so we attributed the cancellation we observed to the HE effect. However, it is apparent that the phenomena under consideration are intrinsically involved with breakaway and so we have begun to model that process.

The dislocation aspects of acoustic harmonic generation were first considered in the series of papers from Brown University that began in 1963 [4]. HE concluded that "In view of the difficulties in separating the lattice and dislocation contributions in the case of the second harmonic, dislocation dynamics are studied more easily through the generation of third harmonics." They studied the response of the vibrating string model, with no breakaway, to a traveling wave. They assumed isotropic elasticity, the nonelastic strain was taken proportional to the area swept out by the moving dislocation, and the nonlinearity responsible for the harmonic generation was associated with the change in character of a dislocation as it bows out. Their results showed a frequency dependence that peaked around the fundamental vibration frequency of the pinned dislocation, a frequency generally well into the megahertz region; but in the kilohertz region their model predicts negligible harmonic generation; this prediction may have led some to discount the appearance of harmonics at low frequencies of vibration.

However, other conditions may lead to a different conclusion. Two aspects are discussed here. First, we consider a standing wave. Using the results of Buck and Thompson [5] for reflection of a pulse, Miller [6] treated a standing wave as a very long pulse and found that the amplitude of the standing wave is increased over that of the traveling wave by a factor of $1/\alpha a$, where α is the attenuation with respect to distance and $a/2$ is the length of the specimen. When the amplitude of the nth harmonic of the traveling wave is proportional to the nth power of the net amplitude of the fundamental, then in the standing wave case the amplitude of the nth harmonic will be enhanced by $(1/\alpha a)^n$, which can be a very large factor.

Second, another model may have a different dependence on frequency. This is seen explicitly in the simple models developed in refs. 7 and 8 which are based on phenomenological representations of dislocation strain that are common in the study of plasticity and in which inertial effects are discounted. For these models, explicit calculation reveals large harmonics, particularly at stresses just above breakaway. For example, Beshers *et al.* supposed that dislocations move only when the effective stress σ is greater than some friction stress σ_i and the velocity is then proportional to $(\sigma-\sigma_i)^2$ but

0925-8388/94/$07.00
SSDI 0925-8388(93)00006-K

with the sign of σ. When $\sigma = 1.2\sigma_i$ the ratio of the third harmonic to the first is -0.82, which is a very substantial contribution. More generally [9], any hysteresis loop may be represented as a Fourier series and the harmonic content will usually be high. These observations suggest that a modeling of the breakaway process itself might be useful.

The material of the experimental studies was in fact armco Fe, that is it had a modest impurity content, and was chosen intentionally to restrain the motion of dislocations until very high vibratory stresses were achieved. For the data discussed here the specimens were in a state of magnetic saturation so that there was no contribution from the magnetomechanical damping as shown by Coronel and Beshers [9]. While we made many observations of harmonics associated with magnetomechanical damping, they were not reproducible, a circumstance we attribute now to a lack of a standard magnetic state; for this paper the point is that the harmonics under discussion were reasonably reproducible (the dip was seen in 10 out of 11 runs) and so we are confident that these data represent the motion of dislocations and not domain walls.

The HE model [3] is based on the vibrating string and is applicable under conditions where there is a fairly free length of dislocation at low strain amplitudes, so that the motion of the dislocations is governed by effective inertia, damping and curvature terms; the resonance behavior of the string is involved and this implies the megahertz region. However, for vibrations in the kilohertz region the vibrating string model, as represented by the amplitude-independent part of the Granato–Lücke (GL) treatment, does not represent the situation. Moreover, we are concerned with the amplitude-dependent hysteretic damping associated with the breakaway and recapture of dislocations moving in a field of point defects considered as minor pinning points, the success of the GL theory. The hysteretic damping is independent of frequency, or very nearly so, and requires a treatment quite different in spirit from that of HE.

2. Breakaway model

In modeling the breakaway dynamics, we consider only the low frequency limit with damping and inertia neglected. We have chosen to start with a rather artificial model that can be evaluated analytically. (It can also be used to validate our numerical schemes.) The dislocation strain is proportional to the area swept out by the moving dislocation, and we represent that area by means of two segments of sine wave with a jump discontinuity in the amplitude of the wave. Because we envision breakaway as occurring on rising stress in each half cycle, followed by retrapping on the falling stress, the discontinuity occurs in each half cycle, positive and negative. Taking $\theta = \omega t$, where ω is the circular frequency of vibration, and t is the time, and ψ as the phase angle at which the jump occurs, we represent the area over the interval $0 \leqslant \theta \leqslant 2\pi$ by:

$$\begin{aligned} \text{Area} &= f \sin\theta,\ 0 \leqslant \theta \leqslant \Psi, \quad \text{and} \quad \pi \leqslant \theta \leqslant \pi + \Psi \\ &= \sin\theta,\ \Psi < \theta \leqslant \pi, \quad \text{and} \quad \pi + \Psi \leqslant \theta \leqslant 2\pi \end{aligned} \tag{1}$$

where f gives the amplitude of the first segment relative to the second which is taken as unit amplitude. Expanding this function in a Fourier series we have formally

$$\text{Area} = \textstyle\sum_{n=0}^{\infty} (A_n \sin n\theta + B_n \cos n\theta) \tag{2}$$

We find that all the even harmonics vanish by symmetry and the odd harmonics are given by

$$A_1 = 1 + F(\sin 2\Psi - 2\Psi) \tag{3}$$

$$B_1 = F(\cos 2\Psi - 1) \tag{4}$$

$$A_n = 2F[S(n+1) - S(n-1)] \tag{5}$$

$$B_n = 2F[C(n+1) - C(n-1)] \tag{6}$$

where $S(n) = (\sin n\Psi)/n$, $C(n) = (\cos n\Psi - 1)/n$, and $F = (1-f)/2\pi$. The amplitude AMP_n and phase ϕ_n of the nth harmonic are given by

$$\text{AMP}_n = (A_n^2 + B_n^2)^{1/2} \tag{7}$$

$$\phi_n = \arctan(-B_n/A_n) \tag{8}$$

We carried out this analysis for $f = 1/2$ and for eight values of $\Psi = p\pi/16$, p running from 1 to 8 so as to cover the range of Ψ from 0 to $\pi/2$. The results of the analysis appear in Table 1. We do not report the results for $\Psi = 0$ because there is no real discontinuity in that case: breakaway occurs instantly. Note also that ϕ_n is independent of f so that our model is not as restrictive as it seems at first.

The amplitude of the effective stress runs up the table; that is, the larger the amplitude of the stress

TABLE 1. Fourier analysis of artificial breakaway, as a function of Ψ, the phase angle of the breakaway event, for integer values of p in $\Psi = p\pi/16$; f is taken as 0.5. AMP_n and ϕ_n are the amplitude and phase of the nth harmonic for each Ψ

Ψ ($\pi/16$)	AMP_1	ϕ_1°	AMP_3	ϕ_3°	AMP_5	ϕ_5°
1	0.99	0.35	6.1×10^{-3}	−67	6.0×10^{-2}	−52
2	0.99	1.3	2.3×10^{-2}	−45	2.2×10^{-2}	−14
3	0.98	2.9	4.9×10^{-2}	−22	4.2×10^{-2}	24
4	0.96	4.8	8.0×10^{-2}	0	5.9×10^{-2}	63
5	0.92	6.8	1.1×10^{-1}	23	6.8×10^{-2}	−75
6	0.88	8.9	1.3×10^{-1}	44	6.7×10^{-2}	−28
7	0.83	10.7	1.5×10^{-1}	68	5.8×10^{-2}	26
8	0.77	12	1.6×10^{-1}	90	5.3×10^{-2}	90

wave, the earlier in the cycle does breakaway occur. For the fundamental, $n=1$, we could calculate the internal friction and the modulus defect if we knew the density of dislocations, but as matters stand we have just calculated the relative defect contribution to the strain, the in-phase and out-of-phase parts being recoverable from the data.

Equations (5) and (6) suggest, and the numerical calculations bear out, that the amplitudes of the harmonics drop off fairly slowly with n. However, in our apparatus we seldom observe harmonics higher than the fifth. This is because of the short length of our specimens, short compared with a wavelength of sound at our frequency (22.5 kHz), in which resonance is achieved by having heavier end pieces on a relatively small (diameter about 2.5 mm) middle section corresponding more to a mass and spring than to the resonant bar. For higher harmonics, the wavelength of the harmonic becomes comparable to the length of the highly stressed central portion and so waves emitted at the opposite ends of the specimen have differing phase when they arrive at the detector and so tend to cancel each other.

Table 1 shows that there are reversals in the sign of the phases as Ψ varies. For ϕ_3, the phase angle at the reversal is $\pi/4$. For ϕ_5 there are three reversals. With a distribution of dislocations with differing stresses for breakaway, which is just what the GL theory contemplates, there will be a distribution of phase angles, positive and negative, and so even with one character (edge or screw or mixed) of dislocation breaking away there should be ample opportunity for a phase cancellation as reported for ϕ_3 in iron [1, 2].

It is also interesting that the extensive data on harmonic generation in brass [8] show a number of dips in the curves of AMP_3 and AMP_5 *vs.* strain amplitude, not as marked as the dip in iron, but definitely there, with often two dips on the curves for AMP_5, but only one, if any, on the curves for AMP_3. Unpublished data in the thesis of M.C. Jon confirm this trend [10].

This new explanation is preferred over the edge/screw phase difference in explaining the dip in AMP_3 in iron because, in fact, the observed cancellation occurs within the first GL segment, rather than exactly at the transition between the segments, or a little above it, as would be expected if approximately equal populations of moving edge and screws were required.

3. Conclusion

This simple model of breakaway seems to have captured the essence of the available evidence at kilohertz frequencies for phase changes and wave cancellations. The phase of the emitted wave is governed by the phase in the vibration at which breakaway occurs. Contributions from change of character with bowout are still to be expected, as are contributions from inertia and damping terms at higher frequencies.

References

1 V.F. Coronel and D.N. Beshers, *J. de Phys., 46* (1985) C10-175.

2 D.N. Beshers and V.F. Coronel, in N. Hessel Andersen, M. Eldrup, N. Hansen, D. Juul Jensen, T. Leffers, H. Lilholt, O.B. Pedersen and B.N. Singh (eds.), *Microstrctural Characterization of Materials by Non-Microscopical Techniques,* Risø National Laboratory, Roskilde, Denmark, 1985, p. 205.

3 A. Hikata and C. Elbaum, *Phys. Rev., 144* (1966) 469.

4 A. Hikata, B.B. Chick and C. Elbaum, *Appl. Phys. Lett., 3* (1963) 195.

5 O. Buck and D.O. Thompson, *Mater. Sci. Eng., 1* (1966) 117.

6 J.G. Miller, *Rev. Sci. Inst., 42* (1971) 537.

7 D.N. Beshers, M.C. Jon, G.M. Beshers and V. Seth, in R.R. Hasiguti and N. Mikoshiba (eds.), *Internal Friction and Ultrasonic Attenuation in Solids,* University of Tokyo Press, Tokyo, 1977, p. 623.

8 M.C. Jon, W.P. Mason and D.N. Beshers, *J. Appl. Phys., 49* (1978) 5871.

9 V.F. Coronel and D.N. Beshers, *J. Appl. Phys., 64* (1988) 2006.

10 M.C. Jon, Ultrasonic study of microplasticity in alpha-brass, *Eng. Sc. D. Thesis*, Columbia University, NY, 1975, unpublished work.